We now turn to the case of designs that are ignorable conditional on covariates. Consider a randomized complete block experiment where treatments are assigned within three sites . This design corresponds to the model

$$g(\mu, \boldsymbol{\tau}, \boldsymbol{\beta}, \mathbf{x_i}, \mathbf{w}_i) = \mu + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \tau_1 w_{1i} + \tau_2 w_{2i} \tag{1}$$

$$y_i \sim \text{normal}(\text{g}(\mu,\boldsymbol{\tau},\boldsymbol{\beta},\mathbf{x_i},\mathbf{w}_i),\sigma^2),$$

to properly reflect how the data were collected. The parameter $\beta_j,\ j = 1, .., 3$, is the change in the overall mean attributable to an observation being taken on site $j$, also called the block or site effect.Thus, we satisfy of missing at random by adding the covariate vector $(\mathbf{x}_i)$ that represents the restriction on randomization imposed by blocking.

Here is the key bit on using subscripts instead of covariates. We can rewrite eq. 1 by defining

$$j = \begin{cases} x_1 = 1: & 1 \\ x_2 = 1: & 2 \\ x_3 = 1: & 3 \end{cases} \tag{2}$$

and letting $\beta_{0j} = \mu + \beta_j$ for each observation $i$. Our model is now more familiar

$$g(\beta, \boldsymbol{\tau}, \mathbf{w}_i) = \beta_{0j} + \tau_1 w_1 + \tau_2 w_2 \tag{3}$$

$$y_{ij} \sim \text{normal}(\text{g}(\beta_{0j},\boldsymbol{\tau},\mathbf{w}_i),\sigma^2),$$

where $y_{ij}$ is the $i^{th}$ observation within in site $j$. Equation 3 means that each site is allowed to have its own mean net mineralization rate in the absence of treatment[1]. Inference on the overall mean rate $(\mu)$ can be obtained by making the model hierarchical as we have done

[1]Equation 1 can also be recast to $y_{ijk} = \mu + \beta_j + \tau_k + \epsilon_{ij},\ \epsilon_{ij} \sim \text{normal}(0, \sigma^2)$, the form taught in most classical texts on experimental design.

earlier(section ):

$$\beta_{0j} \sim \text{normal}(\mu, \varsigma^2). \tag{4}$$