Marginal Distributions

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Slides adapted from Bailey Fosdick, Tom Hobbs, Chris Che-Castaldo, Mary B. Collins

Marginal distributions

In Bayesian analysis, our goal is to compute the joint distribution of the parameters and data:

[parameters | data] = $[\beta_0, \beta_1, \sigma^2, \mid y]$

However, usually want to make inference on a single parameter. We are interested in the marginal (univariate) distribution, e.g.

$$[\beta_0 \mid y]$$

$$[\beta_1 \mid y]$$

$$[\sigma^2 \mid y]$$

Why marginal distributions?

Ex: Discrete joint distribution

Suppose we are studying a species for which births occur in pulses. We observe 100 females and record the number of offspring each produced. Values in the cells of the table are probabilities.

Y = number of offspring

X = age

	1	2	3
1	0.10	0	0
2	0.13	0.12	0.02
3	0.23	0.36	0.04

Ex: Discrete marginal distribution

The probability that a bird produces one or more offspring is the marginal distribution of X.

Y = number of offspring

X = age

	1	2	3	[x]
1	0.10	0	0	0.10
2	0.13	0.12	0.02	0.27
3	0.23	0.36	0.04	0.63

Marginal (dist) of X

Ex: Discrete marginal distribution

The probability that a bird aged one year produces one or more offspring is the sum across row one.

Y = number of offspring

X = age

	1	2	3
1	0.10	0	0
2	0.13	0.12	0.02
3	0.23	0.36	0.04

[x]
0.10
0.27
0.63

Marginal (dist) of X

Ex: Discrete marginal distribution

The probability that a bird produces one, two, or three offspring (regardless of age) is the marginal distribution of Y.

Y = number of offspring

X = age

	1	2	3
1	0.10	0	0
2	0.13	0.12	0.02
3	0.23	0.36	0.04

[x]
0.10
0.27
0.63

Marginal of Y →

[y]	0.46	0.48	0.06

Discrete marginal distributions

$$[x, y] = P(X = x, Y = y)$$ specifies the **joint pmf** of the discrete r.v.s X and Y

$$[x] = \sum_{y} [x, y]$$ specifies the **marginal pmf of x**

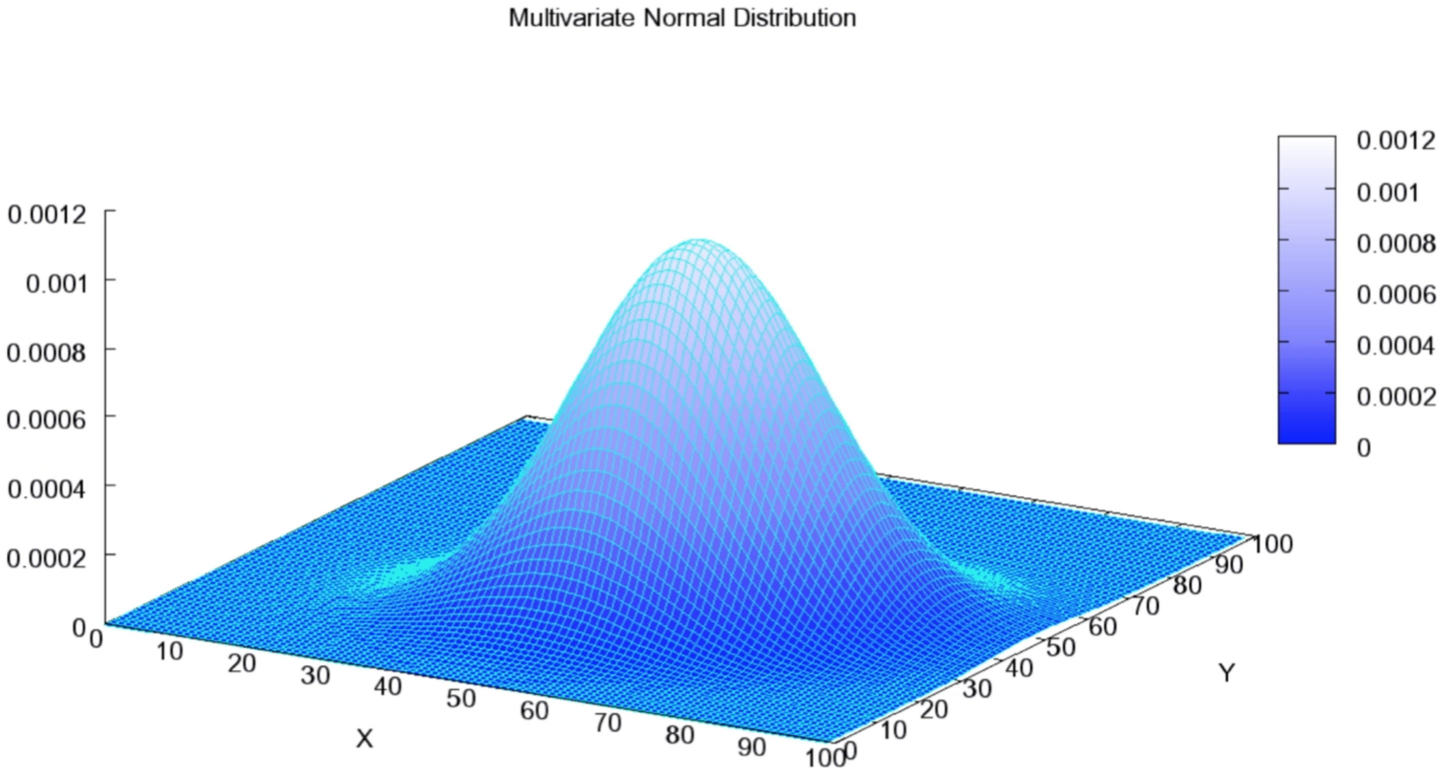
$$[y] = \sum_{x} [x, y]$$ specifies the **marginal pmf of y**

Discrete marginal distributions

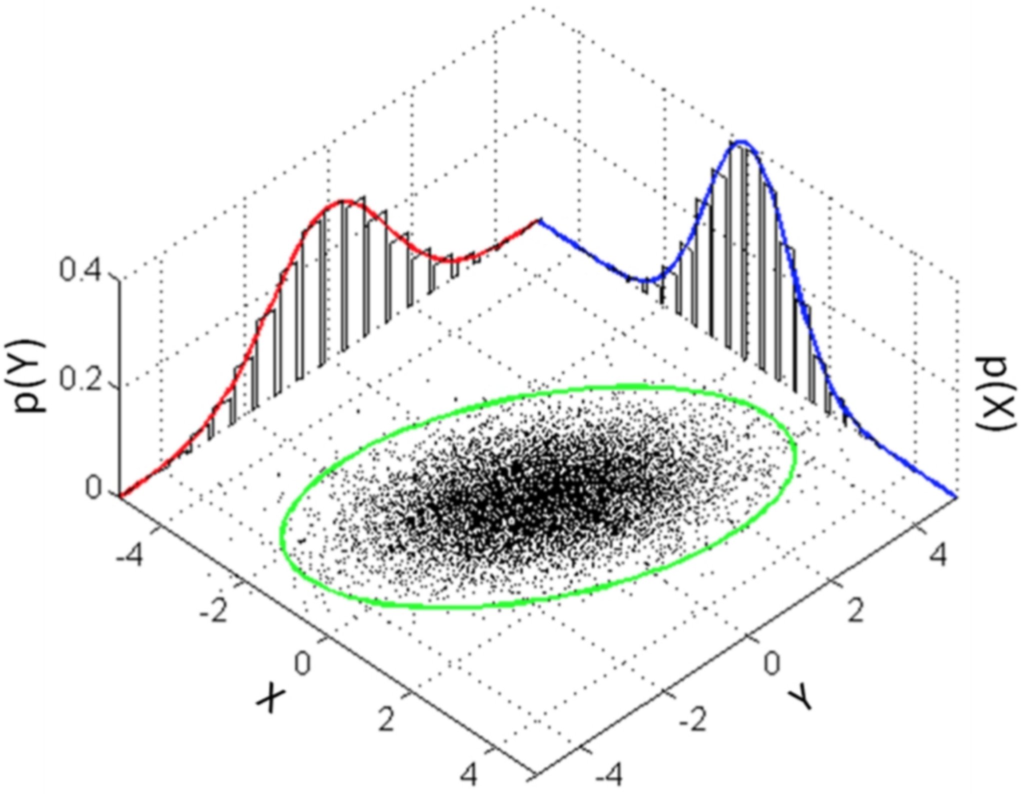
Suppose we have many r.v.s: $[w, x, y, z]$

The marginal distribution of X is $$[x] = \sum_w \sum_y \sum_z [w, x, y, z]$$

Ex: Continuous joint distribution



Ex: Continuous marginal distributions



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Continuous marginal distributions

$$[x, y] = f(x, y)$$

specifies the **joint pdf** of the r.v.s X and Y

$$[x] = \int_{-\infty}^{\infty} [x, y] dy$$

specifies the **marginal pdf of x**

$$[y] = \int_{-\infty}^{\infty} [x, y] dx$$

specifies the **marginal pdf of y**

Marginal distributions (in general)

$$[x, y, z]$$

joint distribution of the r.v.s X, Y, and Z

(suppose X & Y discrete, Z continuous)

$$[x] = \int_{-\infty}^{\infty} \left(\sum_{y} [x, y, z] \right) dz$$

specifies the **marginal pmf of x**